DESIGN OF A HIGH-PERFORMANCED VIBRATOR FOR LINEAR ULTRASONIC MOTORS USING A GENETIC ALGORITHM AND A FINITE ELEMENT ANALYSIS

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ABSTRACT

An optimum design method is proposed for designing the vibrator for the linear ultrasonic motors using genetic algorithms, finite element method and evaluation function based on design conditions. As a result, basic geometry of the vibrator for small-sized and high-performanced linear ultrasonic motors is generated. Dimension is within ten millimeters wide and twenty millimeters long. Obtained geometry of the vibrator satisfies all the design conditions. The obtained geometry and natural vibration modes of the vibrator are so identical that it can not be designed by the trial and error by designers.

1. INTRODUCTION

In recent years, actuators are desired to be smaller-sized and have higher power, in order to construct small-sized and accurate driving systems used for informational machineries and robots. Ultrasonic motors are expected as new small-sized actuators for accurate positioning because of their excellent characteristics including large output power per unit volume, large holding force and large output trust at low speed.

Ultrasonic motors generally consist of a vibrator and a moving element. Natural frequencies of two natural vibration modes of the vibrator must be correspond so that points on the vibrator in contact with the moving element draw an elliptic locus. The moving element in contact with the contact points on the vibrator is driven by frictional force between the moving element and the elliptically moving contact points. In other words, design process of the vibrator is an optimization process for the vibrator's geometry and the natural vibration modes.

A solution space for optimizing the geometry and the natural vibration modes of the vibrator is extremely large. There are many possibilities for the geometry of the vibrator and the natural vibration modes, as long as the contact points on the vibrator move on the desired locus. So, many types of ultrasonic motor were proposed[1]-[6]. However, the basic geometries of the proposed ultrasonic motors are limited as rectangle or cylinder, because it is easy for these simple designs to optimize the geometry and the natural vibration modes, simultaneously. In other words, it is difficult to design the vibrator having complicated geometry and natural vibration modes.

In general, rotational type ultrasonic motors use two vibration modes having similar-geometry, sine and cosine modes. On the other hand, linear type ones use two different vibration modes. Hence, a linear ultrasonic motor is more difficult to design. Because of this, linear ultrasonic motors formerly developed are kept to be designed as simple geometry and are not optimized. The reason for this is that the vibrator is designed due to trial and error.

On the other hand, studies on optimum design for structures using a computer simulation are recently focused on[7][8][9]. However, most of them aim for simple problems such as a sizing problem that only optimize a total weight. Optimum design using simulation has not been applied to dynamic design problems such as a vibrator design whose geometry is comparatively complex.

The authors propose a geometry optimization method for an ultrasonic motor's vibrator utilizing a simple genetic algorithm (GA)[10][11] and a finite element method (FEM). The driving principle and the design conditions of the linear ultrasonic motors are described at first. Then, an optimum design method for basic geometry of vibrator for linear ultrasonic motor, utilizing GA and FEM, is proposed. Finally, the proposed design method is applied to a linear ultrasonic motor. It is shown that the basic geometry of the vibrator for the linear ultrasonic motor satisfying the design conditions is designed using the proposed method.

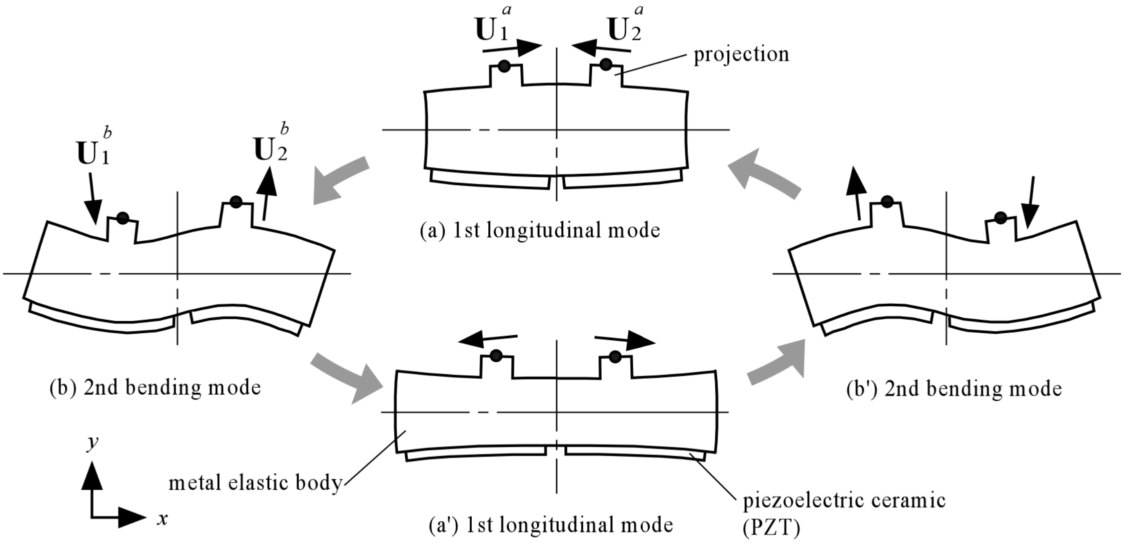


Fig. 1 Driving principle of former linear ultrasonic motor

2. DESIGNING CONDITIONS OF LINEAR ULTRASONIC MOTORS

2.1 Driving Principle

A standing wave type linear ultrasonic motor is taken as an object of this study. Figure 1 shows an example of basic driving principle of the linear ultrasonic motors developed before[4][5][6]. The vibrator consists of metal elastic body with PZTs located underneath the elastic body. Two projections are located on the vibrator where a slider contacts. The geometry of the vibrator is designed so that the natural frequencies of two vibration modes, i.e. a first longitudinal mode (Fig. 1(a) and (a')) and a second bending mode (Fig. 1(b) and (b')), almost correspond. Points on the projections move from left to right as shown in Fig. 1 (a) and (a') as vectors $\mathbf{U}_i^a$ (i=1,2) when the first longitudinal mode is excited. They move up and down as shown in Fig. 1 (b) and (b') as vectors $\mathbf{U}_i^b$ (i=1,2) when the second bending modes is excited. When the first longitudinal mode and the second bending mode are combined with phase difference of quarter cycle, the two projections draw a clockwise elliptic locus as shown in Fig. 2 when time passes from (a), (b), (a') to (b') because the two vectors, $\mathbf{U}_i^a$ and $\mathbf{U}_i^b$, are added. Then, the slider in contact with the two projections is moved to the right in order. When the sign of the phase difference between the two natural vibration modes is changed, the rotating direction of the projections is reversed to counterclockwise. Then, the moving direction of the slider is also reversed.

2.2 Design Conditions

A design process of the vibrator for high-performanced linear ultrasonic motor is to determine the geometry of the vibrator and to select its natural vibration modes. The required design conditions for the vibrator are as follows:

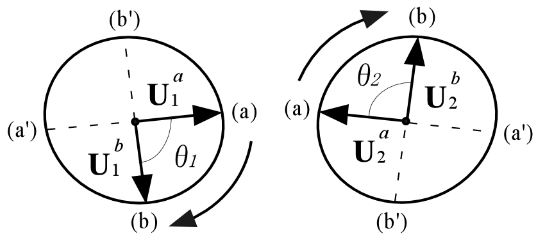


Fig. 2 Displacement vectors and elliptic motion at the contact points on vibrator

(1) Frequencies of the combined two natural vibration modes correspond.
(2) The vectors, $\mathbf{U}_i^a$ and $\mathbf{U}_i^b$ (i=1,2), cross at right angles each other.
(3) The rotational directions of the elliptic motions of the vectors are the same for different contact points.
(4) The angle between the vectors, $\mathbf{U}_1^a$ and $\mathbf{U}_2^a$ (*cf.* Fig. 1), is π radians.
(5) The amplitude at the contact points is relatively large.

The condition (1) is required because it can't make the cyclic elliptic locus if the two frequencies are different. The condition (2) is for approximating the elliptic locus to the circle. If the locus drawn by the contact points is inclined ellipse, driving efficiency of the motor is decreased. The condition (3) means that the direction of the slider can't be fixed and the driving efficiency of the motor is decreased if the rotational directions of the contact points differs each other. The condition (4) is for approximating the interval of contact between the vibrator and the slider to regular interval. If it is irregular, the efficiency of the motor and the robustness for the disturbance decrease. The condition (5) is for reducing the internal loss, and for enlarging the efficiency. If the amplitude at any part except for the contact points is large, the internal loss increases.

It is difficult to design the vibrator which satisfies all the design conditions simultaneously by the general design method, designers' trial and error, because the design conditions have a large non-linear characteristics, and the design freedom is large. So, the former ultrasonic motor shown in Fig. 1 and Fig. 2 satisfies a few conditions listed above. What is worse, it is difficult to produce a small-sized and high-performanced vibrator.

3. DESIGN METHOD

3.1 Evaluation Method for Geometry

Natural frequencies and natural vibration modes are calculated by the eigenvalue analysis using the finite element method. The two natural vibrations selected optionally are evaluated using an evaluation function E. The evaluation function E is defined by each evaluation functions for each design condition defined as follows when the two natural vibration modes, mode a and b, are selected by m modes and the two contact points is located on the vibrator:

$$E = \alpha \times E_1 + \beta \times E_2 + \gamma \times E_3 + \delta \times E_4 \qquad (1)$$

$$E_1 = \frac{f_b - f_a}{f_a} \quad (f_a \leq f_b) \qquad (2)$$

$$E_2 = 1 - \left| 1 - \frac{1}{2}\sum_{i=1}^{2}(1 - \sin\theta_i) \right| \qquad (3)$$

$$E_3 = \frac{1}{2}\left\{ \frac{1}{2}\left(1 - \cos\phi_{12}^{a}\right) + \frac{1}{2}\left(1 - \cos\phi_{12}^{b}\right) \right\} \qquad (4)$$

$$E_4 = \frac{1}{2}\sum_{i=1}^{2}\frac{1}{2}\left\{ \left(1 - \frac{\left\|\mathbf{U}_i^a\right\|}{\left\|\mathbf{U}_{\max}^a\right\|}\right) + \left(1 - \frac{\left\|\mathbf{U}_i^b\right\|}{\left\|\mathbf{U}_{\max}^b\right\|}\right) \right\} \qquad (5)$$

where, α, β, γ and δ are weights for each design conditions. f_a and f_b represent the frequency of mode a and mode b, respectively, provide that f_b is larger than f_a. $\mathbf{U}_i^a$ shows the vector of the mode a at the contact point i, $\mathbf{U}_{max}^a$ shows a maximum displacement of the mode a. θ_i represents the angle between $\mathbf{U}_i^a$ and $\mathbf{U}_i^b$ as shown in Fig. 2. ϕ^l_{12} is calculated by $\mathbf{U}_1^l$ and $\mathbf{U}_2^l$ (l=a,b) as shown in Fig. 3. The function E_1 is for the condition (1), the E_2 is for the (2) and the (3), the E_3 is for the (4), and the E_4 is for the (5). Functions, E_1 to E_5, converge to zero, when the vibrator satisfies each design conditions.

The evaluated value of the geometry of the vibrator is obtained when the optional two vibration modes are

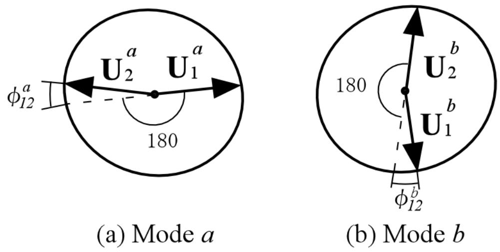


Fig.3 Displacement vectors in each mode at the contact points on vibrator

selected. The evaluation values for all combinations of modes are calculated as $E_{\min}$ by the function E as follow:

$$E_{\min} = \min\{E \mid {}_2C_m\} \qquad (6)$$

Finally, the minimum evaluation value $E_{\min}$ is obtained for evaluating the geometry.

3.2 Genetic Algorithm

Space of parameters for designing geometry of a vibrator for the linear ultrasonic motors has large non-linear characteristics and has large solution space. The solution space has many local optimum points. Global sampling method for this discontinuous solution space is suitable. For this, GA is used in this study.

A flow chart of the analysis is shown in Fig. 4. The fundamental flow is described as follows. First, individuals consist of n populations, which express the vibrators, are made as binary code shown as genotype in Fig. 4 from random numbers. Each population expressed in genotype is decoded to two dimensional finite element models as phenotype as shown in Fig. 4. Next, natural frequencies and natural modes of each FE model are analyzed using eigenvalue analysis of finite element method. Then, the result of finite element analysis is evaluated using the evaluation function $E_{\min}$. $E_{\min}$ is also evaluated whether it is in a settled convergence range. If the right result is not acquired, the genetic operations known as selection, crossover and mutation are applied to make new individuals of genotypes. By repeating the above calculation, a profitable basic geometry of vibrator for linear ultrasonic motor is acquired.

Each operation, i.e., decoding from the genotype to the phenotype, genetic operation and a repairing method of the model, are described in detail as follows:

(a) Genotype and Phenotype

The genotype is shown as a vector of binary numbers, 0 and 1, as shown in Fig. 4. On the other hand, the phenotype of the vibrators is expressed as a matrix constructed by considering existence or nonexistence of square plane strain finite elements. When the gene is 1, the element exists as shown by black rectangular in Fig. 4. When it is 0, element does not exist as shown by white rectangular in Fig. 4.

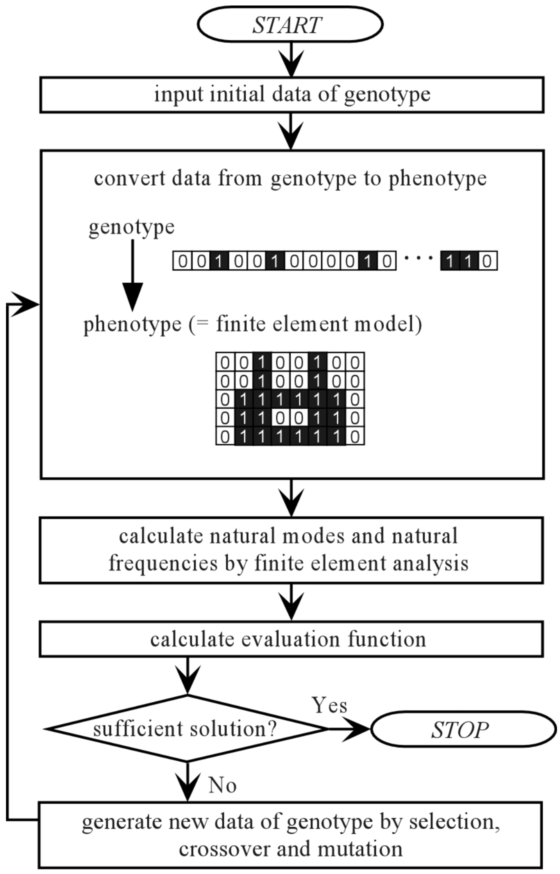


Fig. 4 Flow chart of analysis

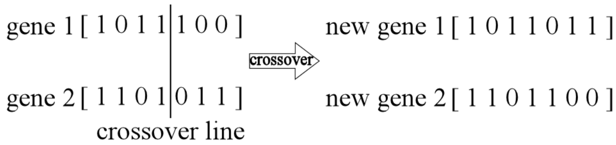


Fig. 5 Example of crossover operation

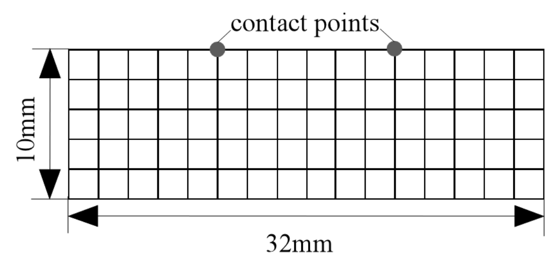


Fig. 6 Range for generating finite elements

The vector of genotype is transformed from the matrix of phenotype by extending from the upper-left to lower right. Actually, half side of the phenotype model is used because the geometry of vibrator for a linear ultrasonic motor is symmetry with respect to the central vertical line.

(b) Genetic Operations

New genotypes are generated using operations. Each genetic operation is described as follows.

As for the selection, a roulette selection, which selects individuals according to their evaluation values, and a elitist preserving selection, which select a most suitable individual, are used. The methods are generally-used simple GA.

As for the crossover, a one-point crossover method, so called a simple crossover, is used. Two individuals optionally selected are changed using the crossover as shown in Fig. 5, if a random number is within crossover rate.

As for the mutation, a simple method is used, in which the gene is changed from 1 to 0 or from 0 to 1 within a mutation rate.

(c) Repair for Finite Element Model

When a gene operated using crossover and mutation is decoded to the phenotype, there can be some cases in which the finite element model is not suitable for the geometry of the vibrator.

One case is that the model is separated into several parts. In such a case, the finite element analysis is not applied to the model and the individual is considered to have a low value for evaluation function.

The other case is that the model has some non-existing elements inside the vibrator as shown in Fig. 4. Because these types of vibrator are difficult to be produced, the finite element model is repaired as follows: The non-existing elements inside the vibrator are converted to existing elements, so that the finite element model suitable for the geometry of the vibrator is reproduced.

4. RESULT OF ANALYSIS

4.1 Conditions of Analysis

The weights, α, β, γ and δ, are set to 0.25. All the design conditions are equally considered.

Number of individuals *n*, crossover rate and mutation rate are settled to be 50, 0.5 and 0.1, respectively. These numbers are general in GA.

A second dimensional finite element model within a square of 10 mm $\times$ 32 mm is adopted, and the size of each element is square of 2 mm $\times$ 2 mm, as shown in Fig. 6. Hence, the number of elements and genes are 80 and 40, respectively. Maximum size of the generated vibrator is almost the same size as the vibrator developed before[4]. Material properties of the brass are applied for each element. Young's modulus is 1.04×10^{11} N/m^2, poison's ratio is 0.33 and mass density is 7.79×10^3 kg/m^3. Free boundary condition is selected for FE analysis. Ten smallest natural vibration modes are calculated. The two contact points are located in the model at points shown in Fig. 6. FE code MARC is used for the analysis.

The calculation is ended when the best individual does not change for over two hundred generations.

4.2 Results

Convergence history of evaluation value $E_{\min}$ of the best individual is shown in Fig. 7. From the result, it is seen that the best evaluation value decreases when generation increases. Geometry of the best individual is shown in Fig. 8. Selected two natural vibration modes are shown in Fig. 9. The evaluation values of the vibrator for each design condition and the characteristics of vibration are shown in Table 1.

5. DISCUSSION

5.1 Characteristics of Result

The evaluation value E_1 shown in Table 1 means that the rate of the difference between frequencies, f_a and f_b, for the frequency of mode *a* is within 1 %. The value E_2 shows that the angle between the vectors, $\mathbf{U}_i^a$ and $\mathbf{U}_i^b$ (i=1,2) (*cf.* Fig. 2), is 79.6 radians. The value E_3 shows that the difference in angle between π radians and the angle between the vectors, $\mathbf{U}_1^l$ and $\mathbf{U}_2^l$ ($l=a,b$) (*cf.* Fig. 3), is 22.6 radians. The E_4 shows that rate of the amplitude at the contact points for the maximum amplitude is 89.0 %. We can say that these characteristics of vibrator are suitable for the vibrator for linear ultrasonic motors. In other words, the obtained geometry of vibrator for the linear ultrasonic motor satisfies all the design conditions. From the above discussion, the appropriateness of the proposed optimum design method is confirmed.

The obtained geometry is complicated and is not able to be designed by trial and error applied before. It shows the effectiveness of this method.

The natural vibration modes selected from the analysis are different from the one before.

Number of FE elements is small in this study. It may be needed to design its details, using the finite element model having larger number of elements to realize the vibrator.

5.2 Comparison with Former Linear USMs

The obtained geometry of the vibrator shown in Fig. 4 is compared with the formerly developed vibrator[4]. The finite element model of the former vibrator and the two contact points are shown in Fig. 10. The vibration modes are shown in Fig. 11. The characteristics of the vibration and the evaluation values for the design conditions are shown in Table 2.

From the results, characteristics of the vibrator designed using the proposed optimum design method is much suitable as vibrator for ultrasonic motors. Especially, it is smaller in size. However, the evaluation values of the obtained vibrator for the design conditions (2) and (3) are larger than those for the former vibrator. So, it is needed to design a higher-performanced vibrator in details.

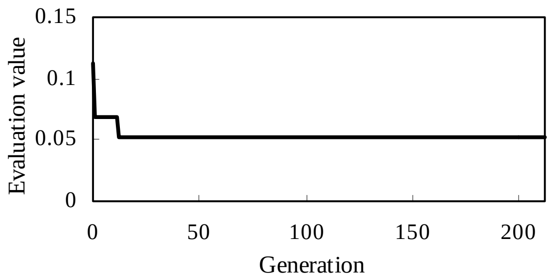


Fig. 7 Convergence histry of evaluation value $E_{\min}$

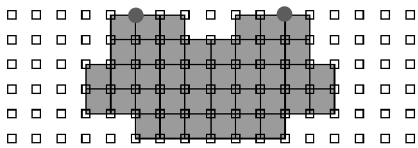
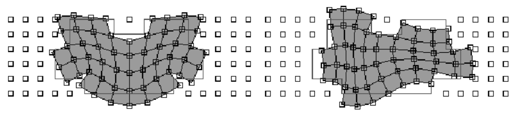


Fig. 8 Geometry of generated vibrator



(a) mode *a* (b) mode *b*

Fig. 9 Selected two natural vibration modes

Table 1 Evaluation value for the generated vibrator

Evaluation function	Evaluation value
Frequency of mode *a* [Hz]	164900
Frequency of mode *b* [Hz]	166200
E_1	0.007884
E_2	0.016552
E_3	0.076558
E_4	0.109841

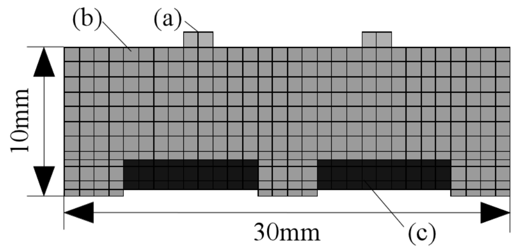
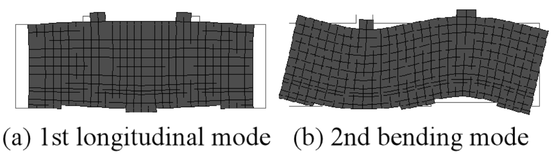


Fig. 10 Geometry of former vibrator
(a) Contact point(projection)
(b) Metal elastic body
(c) Piezoelectric ceramic(PZT)



(a) 1st longitudinal mode (b) 2nd bending mode

Fig. 11 Two natural vibration modes of former vibrator

Table 2 Evaluation value for former vibrator

Evaluation function	Evaluation value
Frequency of mode *a* [Hz]	53000
Frequency of mode *b* [Hz]	51200
E_1	0.035156
E_2	0.002536
E_3	0.168813
E_4	0.500578

It is shown that the geometry of the vibrator, which satisfies the design condition, can be produced using the proposed method.

However, several improvements are needed. First, the simple GA may not be the best way to design the vibrator of the linear ultrasonic motors. So, the decoding method including L-system and GP should be conducted to obtain optimum design within a small generation. Second, the number of element of the FE model is small in this study. It is difficult to design the vibrator in detail. We should try to make more precise FE model. Next, the result is not actually confirmed by making the vibrator. We should make the linear ultrasonic motor using the obtained geometry of vibrator in the future study.

6. CONCLUSION

A design method for the basic geometry of the vibrator for the linear ultrasonic motors is proposed. The evaluation function is defined based on the design conditions calculated from the results of the finite element analysis to evaluate the geometry of vibrator. A genetic algorithm is used as an optimization method.

Then, the basic geometry of the vibrator for the linear ultrasonic motor is designed using the proposed method. Obtained geometry of the vibrator satisfies the design conditions. The obtained geometry and natural vibration modes of the vibrator are so identical that it can not be designed by the trial and error by designers.

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